Ladder operator formalisms and generally deformed oscillator algebraic structures of quantum states in Fock space

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Abstract

We show that various kinds of one-photon quantum states studied in the field of quantum optics admit ladder operator formalism and bear generally deformed oscillator algebraic structure. The two-photon case is also considered. We obtain the ladder operator formalisms of two general states defined in the even/odd Fock space. The two-photon states also bear generally deformed oscillator algebraic structure. Some interesting examples of one-photon and two-photon quantum states are given.

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I. INTRODUCTION

The interesting quantum states of the radiation field such as coherent states(CSs)[1], squeezed states(SSs)[2], binomial states(BSs)[3] and negative binomial states(NBSs)[4] have been studied in detail in the literature. The well known CS has many applications in both the field of quantum optics and the field of condense matter. The CS is the eigenvector of the boson annihilation operator, while the SS is the eigenvector of a linear combination of the boson annihilation and creation operators. The BS interpolates between two fundamental states, the CS and the Fock state, and reduces to them in two different limits[3]. It partakes the properties of the CS and the Fock state. The photon number distribution of the BS is binomial distribution in the probability theory, while the photon number distribution of the NBS is negative binomial distribution. The NBS can also reduce to the CS in a certain limit. In the sense that the BS and the NBS can reduce to the CS, these two states can be regarded as the generalizations of the CS.

All the states mentioned above have certain algebraic structures. The CS bears the Heisenberg-Weyl algebraic structure and the SS bears the su(1,1) Lie algebraic structure. It is found that the BS and the NBS also bear Lie algebraic structures. The algebra involved in the BS[5] is the su(2) Lie algebra via Holstein-Primakoff realization[6] and that involved in the NBS[7] is the su(1,1) Lie algebra via Holstein-Promakoff realization. It is well known that the CS and SS can be written as the displacement operator formalisms. The BS and NBS also admit displacement operator formalisms. As a generalization of the BS and a generalization of both the BS and the NBS, the hypergeometric state(HGS)[8] and P$\acute{o}$lya state(PS)[9] are introduced by Fu et al, respectively. The HGS reduce to BS in a certain limit and the NHGS reduces to the BS and the NBS in two different limits. The photon number distribution of the HGS is the hypergeometric distribution in probability theory and that of the NHGS is the P$\acute{o}$lya distribution. The algebraic structure of the HGS and PS are the well investigated generally deformed oscillator (GDO) algebra[10]. Recently Roy et al. and

Fan et al. also introduce two kinds of quantum states which are claimed to be BS-NBS intermediate states[11,12]. But we have proved that the two kinds of states are identical to the PSs[13].

The GDO first appeared in Heisenberg's theory of nonlinear spinor dynamics[10]. Many physical systems are found to enjoy the GDO symmetry[14]. The GDO is an associate algebra over the complex number field C with generators $A^+, A, \mathcal{N}$ and the unit 1 satisfying

$$[\mathcal{N}, A^+] = A^+, [\mathcal{N}, A] = -A, AA^+ = F(\mathcal{N}+1), A^+A = F(\mathcal{N}), \tag{1}$$

where the Hermitian non-negative function F is called the structure function. It should satisfy the condition $F(0) = 0$ in order to have Fock representation. In this paper we try to find ladder operator formalisms and the GDO algebraic structures of various quantum states including both one-photon and two-photon cases.

II. ONE-PHOTON QUANTUM STATES

A. Quantum states as a linear combination of first finite Fock states

We first consider the state $|x, M\rangle$ defined as a linear combination of first finite Fock states in an $(M+1)$-dimensional subspace of the Fock space

$$|x, M\rangle = \sum_{n=0}^{M} C(n, x, M)|n\rangle, \tag{2}$$

where x denotes parameters of the state, M is a non-negative integer and $|n\rangle$ is the usual Fock state. We assume the coefficients in this paper are all non-zero. One typical example of the state $|x, M\rangle$ is the BS. Next we want to find the ladder operator formalism of the state $|x, M\rangle$.

Let the operator $f(\hat{N})a$ act on the state and we get

$$f(\hat{N})a|x, M\rangle = \sum_{n=0}^{M-1} f(n)C(n+1, x, M)\sqrt{n+1}|n\rangle, \tag{3}$$

where $f(\hat{N})$ is a nonlinear function of the operator $\hat{N} = a^+a$. Here a^+ and a are the creation and annihilation operators of the radiation field, respectively. If we choose

$$f(\hat{N}) = \frac{C(\hat{N}, x, M-1)}{\sqrt{\hat{N}+1}C(\hat{N}+1, x, M)}, \tag{4}$$

Eq.(3) becomes

$$f(\hat{N})a|x, M\rangle = |x, M-1\rangle. \tag{5}$$

The operator $f(\hat{N})a$ transforms the state $|x, M\rangle$ to the state $|x, M-1\rangle$. The key point is that another operator $g(\hat{N})\sqrt{M-\hat{N}}$ which can make this transformation is found.

It is easy to evaluate that

$$g(\hat{N})\sqrt{M-\hat{N}}|x, M\rangle = |x, M-1\rangle, \tag{6}$$

where

$$g(\hat{N}) = \frac{C(\hat{N}, x, M-1)}{\sqrt{M-\hat{N}}C(\hat{N}, x, M)}. \tag{7}$$

The operator $\sqrt{M-\hat{N}}$ removes the number state $|M\rangle$ from the state $|x, M\rangle$ and the operator $g(\hat{N})$ makes the removed state be the state $|x, M-1\rangle$. Note that the operator $g(\hat{N})\sqrt{M-\hat{N}}$ is not equal to the operator $C(\hat{N}, x, M-1)/C(\hat{N}, x, M)$ since the operator $1/\sqrt{M-\hat{N}}$ is defined in the $M-$dimensional subspace instead of $(M+1)-$dimensional subspace.

Combining Eqs.(5) and (6) leads to

$$f(\hat{N})a|x, M\rangle = g(\hat{N})\sqrt{M-\hat{N}}|x, M\rangle. \tag{8}$$

Substituting Eqs.(4) and (7) into the above equation , we obtain the ladder operator formalism of the state $|x, M\rangle$ as

$$[\hat{N} + \frac{(M-\hat{N})C(\hat{N}, x, M)}{\sqrt{\hat{N}+1}C(\hat{N}+1, x, M)}a]|x, M\rangle = M|x, M\rangle. \tag{9}$$

Now let us examine the algebraic structure involved in the above equation. Define $\mathcal{A}$ as an associate algebra with generators

$$\hat{N}, A_M^- = \frac{(M-\hat{N})C(\hat{N},x,M)}{\sqrt{\hat{N}+1}C(\hat{N}+1,x,M)}a, A_M^+ = (A_M^-)^\dagger. \tag{10}$$

Then it is easy to verify that these operators satisfy the commutation relations

$$[\hat{N}, A_M^\pm] = \pm A_M^\pm, \ A_M^+ A_M^- = F(\hat{N}), \ A_M^- A_M^+ = F(\hat{N}+1), \tag{11}$$

where the function

$$F(\hat{N}) = (M-\hat{N}+1)^2 \frac{C^2(\hat{N}-1,x,M)}{C^2(\hat{N},x,M)}. \tag{12}$$

In comparison with Eq.(1), this algebra $\mathcal{A}$ is nothing but the GDO algebra with the structure function $F(\hat{N})$. In terms of the generators of the algebra $\mathcal{A}$, Eq.(9) can be rewritten as

$$(\hat{N}+A_M^-)|x,M\rangle = M|x,M\rangle. \tag{13}$$

Below we study several interesting special cases of the state $|x,M\rangle$.

1. The BS is defined as[3]

$$|\eta,M\rangle = \sum_{n=0}^{M}\left[\binom{M}{n}\eta^n(1-\eta)^{M-n}\right]^{1/2}|n\rangle. \tag{14}$$

Here η is a real parameter satisfying $0<\eta<1$. From Eq.(9) and (14), we get the ladder operator formalism of the BS

$$[\hat{N}+\sqrt{(1-\eta)/\eta}\sqrt{(M-\hat{N})}a]|\eta,M\rangle_{BS} = M|\eta,M\rangle_{BS}, \tag{15}$$

which is identical to that obtained in Ref.[5].

2. As a one-parameter generalization of the BS, the HGS is given by[8]

$$|L,M,\eta\rangle_{HGS} = \sum_{n=0}^{M}\left[\binom{L\eta}{n}\binom{L\bar{\eta}}{M-n}\right]^{1/2}\binom{L}{M}^{-1/2}|n\rangle, \tag{16}$$

where $\bar{\eta}=1-\eta$, L is a real number satisfying $L \geq \max\{M\eta^{-1}, M\bar{\eta}^{-1}\}$, and

$$\binom{x}{n} = \frac{x(x-1)...(x-n+1)}{n!}, \binom{x}{0} \equiv 1. \tag{17}$$

From the above equation and Eq.(9), we obtain

$$[\hat{N}+(\frac{L\bar{\eta}-M+\hat{N}+1}{L\eta-\hat{N}})^{1/2}\sqrt{(M-\hat{N})}a]|L,M,\eta\rangle_{HGS}=M|L,M,\eta\rangle_{HGS}, \quad (18)$$

which is the ladder operator formalism of the HGS.

3.As a BS-NBS intermediate state, the PS is introduced as[9]

$$|\eta,\gamma,M\rangle_{PS}=\sum_{n=0}^{M}P_n^M(\gamma,\eta)|n\rangle, \quad (19)$$

where

$$\begin{aligned} P_n^M(\gamma,\eta) \;=\; & \binom{M}{n}^{1/2}\{\prod_{k=1}^{n}[\eta+(k-1)\gamma]\}^{1/2}\{\prod_{k=1}^{M-n}[\bar{\eta}+(k-1)\gamma]\}^{1/2} \\ & \{\prod_{k=1}^{M}[1+(k-1)\gamma]\}^{-1/2} \end{aligned} \quad (20)$$

and $\gamma>0$ is a real constant.

The ladder operator formalism of the PS is obtained as

$$[\hat{N}+(\frac{\bar{\eta}+(M+\hat{N}-1)\gamma}{\eta+\hat{N}\gamma})^{1/2}\sqrt{(M-\hat{N})}a]|\eta,\gamma,M\rangle_{PS}=M|\eta,\gamma,M\rangle_{PS}. \quad (21)$$

4. As a reference state of detecting the phase of a quantum state, Barnett and Pegg introduce a reciprocal binomial state(RBS)[15]

$$|\theta,M\rangle_{RBS}=\left(1/\sum_{n=0}^{M}\binom{M}{n}^{-1}\right)\sum_{n=0}^{M}\binom{M}{n}^{-1/2}\exp(in\theta)|n\rangle. \quad (22)$$

The ladder operator formalism is directly obtained as

$$[\hat{N}+\frac{M-\hat{N}}{\hat{N}+1}\exp(-i\theta)\sqrt{(M-\hat{N})}a]|\theta,M\rangle_{RBS}=M|\theta,M\rangle_{RBS}. \quad (23)$$

5.Pegg and Barnett[16]defined the Hermitian phase operator on a finite-dimensional state space, which make it possible to study the phase properties of quantum states of the single mode of the electromagnetic field. The Pegg-Barnett phase states(PBPS) $|\theta_m\rangle$ can be defined as

$$|\theta_m,M\rangle_{PBPS}=\frac{1}{(M+1)^{1/2}}\sum_{n=0}^{M}\exp(in\theta_m)|n\rangle \quad (24)$$

The phase states in Eq.(24) form an orthonormal set provided that we have

$$\theta_m = \theta_0 + 2\pi m/(s+1), m = 0, 1, ..., s, \tag{25}$$

where θ_0 is an arbitrary reference phase.

We give the ladder operator formalism of the PBPS as

$$[\hat{N} + \frac{M-\hat{N}}{\sqrt{\hat{N}+1}} \exp(-i\theta_m)a]|\theta_m, M\rangle_{PBPS} = M|\theta_m, M\rangle_{PBPS}. \tag{26}$$

6. The generalized geometric state(GGSs) is defined as[17]

$$|Y, M\rangle_{GGS} = \left(\frac{1-|Y|}{1-|Y|^{M+1}}\right)^{1/2} \sum_{n=0}^{M} Y^{n/2}|n\rangle \tag{27}$$

where Y is a complex parameter.

The ladder operator formalism of the GGS is directly given as

$$[\hat{N} + \frac{M-\hat{N}}{\sqrt{Y}\sqrt{\hat{N}+1}} a]|Y, M\rangle_{GGS} = M|Y, M\rangle_{GGS}. \tag{28}$$

We also give the corresponding structure functions of the above six quantum states, the BS, HGS, PS, RBS, PBPS and GGS as follows:

$$\begin{aligned}
F_{BS}(N) &= (M-N+1)^3 \frac{1-\eta}{\eta}, \\
F_{HGS}(N) &= (M-N+1)^3 \frac{L(1-\eta)-M+N}{L\eta-N+1}, \\
F_{PS}(N) &= (M-N+1)^3 \frac{(1-\eta)+(M+N-2)\gamma}{\eta+(N-1)\gamma}, \\
F_{RBS}(N) &= \exp(-2i\theta)(M-N+1)^5/N^2, \\
F_{PBPS}(N) &= \exp(-2i\theta_m)(M-N+1)^4/N, \\
F_{GGS}(N) &= (M-N+1)^4/(YN).
\end{aligned} \tag{29}$$

B. Quantum states without first finite Fock states

We further investigate the state $|x, M\rangle^-$ defined as

$$|x, M\rangle^- = \sum_{n=M}^{\infty} D(n, x, M)|n\rangle. \tag{30}$$

The state has no first finite Fock states. One type of this state is the so-called photon-added quantum state[18]

$$|\psi, M\rangle = N_M a^{\dagger M}|\psi\rangle, \tag{31}$$

where $|\psi\rangle$ may be an arbitrary quantum state

$$|\psi\rangle = \sum_{n=0}^{\infty} C(n,x)|n\rangle, \tag{32}$$

and N_M is a normalization constant. For the first time, the photon-added states were introduced by Agarwal and Tara as photon-added coherent states(PACSs)[18].

The key step is to let the operators $f(\hat{N})a^\dagger$ and $g(\hat{N})\sqrt{\hat{N}-M}$ act on the state $|x, M\rangle^-$, and this leads to

$$\begin{aligned} f(\hat{N})a^\dagger|x, M\rangle^- &= |x, M+1\rangle^-, & (33)\\ g(\hat{N})\sqrt{\hat{N}-M}|x, M\rangle^- &= |x, M+1\rangle^-, & (34)\end{aligned}$$

where

$$\begin{aligned} f(\hat{N}) &= \frac{D(\hat{N}, x, M+1)}{\sqrt{\hat{N}}D(\hat{N}-1, x, M)}, & (35)\\ g(\hat{N}) &= \frac{D(\hat{N}, x, M+1)}{\sqrt{\hat{N}-M}D(\hat{N}, x, M)}. & (36)\end{aligned}$$

The operators used here, $f(\hat{N})a^\dagger$ and $g(\hat{N})\sqrt{\hat{N}-M}$, are different from those in deriving the ladder operator formalism of the state $|x, M\rangle$. By acting of the two operators on the state $|x, M\rangle^-$, we can transform the state $|x, M\rangle^-$ to $|x, M+1\rangle^-$.

From Eqs.(33)-(36), we get

$$[\hat{N} - \frac{(\hat{N}-M)D(\hat{N}, x, M)}{\sqrt{\hat{N}}D(\hat{N}-1, x, M)}a^\dagger]|x, M\rangle^- = M|x, M\rangle^-. \tag{37}$$

This is the ladder operator formalism of the state $|x, M\rangle^-$ in terms of the operators $\hat{N}$ and $a^\dagger$. By multiplying the annihilation operator a on the above equation from left, it can be written in terms of the operators $\hat{N}$ and a

$$(\hat{N}+1-M)a|x, M\rangle^- = \frac{(\hat{N}+1-M)\sqrt{\hat{N}+1}D(\hat{N}+1, x, M)}{D(\hat{N}, x, M)}|x, M\rangle^- \tag{38}$$

As a special case of the state $|x,M\rangle^-$, the photon-added state(Eq.(31)) can be expanded as

$$|\psi,M\rangle = N_M \sum_{n=M}^{\infty} C(n-M,x)[n!/(n-M)!]^{1/2}|n\rangle. \tag{39}$$

From the above equation and Eq.(37), we obtain the ladder operator formalism of the photon-added state as

$$[\hat{N} - \frac{C(\hat{N}-M,x)}{C(\hat{N}-M-1,x)}\sqrt{\hat{N}-M}a^\dagger]|\psi,M\rangle = M|\psi,M\rangle. \tag{40}$$

The above equation can be written in terms of the operators $\hat{N}$ and a,

$$(\hat{N}+1-M)a|\psi,M\rangle = \frac{C(\hat{N}+1-M,x)}{C(\hat{N}-M,x)}\sqrt{\hat{N}+1-M}(\hat{N}+1)]|\psi,M\rangle. \tag{41}$$

The algebra involved in the state $|x,M\rangle^-$ is also GDO algebra $\mathcal{B}$. The generators are

$$\hat{N}, B_M^+ = \frac{(\hat{N}-M)D(\hat{N},x,M)}{\sqrt{\hat{N}}D(\hat{N}-1,x,M)}a^\dagger, B_M^- = (B_M^+)^\dagger. \tag{42}$$

They satisfy the commutation relations

$$[\hat{N}, B_M^\pm] = \pm B_M^\pm, B_M^+ B_M^- = G(\hat{N}), B_M^- B_M^+ = G(\hat{N}+1) \tag{43}$$

with the structure function

$$G(\hat{N}) = (\hat{N}-M)^2 \frac{D^2(\hat{N},x,M)}{D^2(\hat{N}-1,x,M)}. \tag{44}$$

In terms of the generators of the algebra $\mathcal{B}$, Eq.(37) becomes

$$[\hat{N} - B_M^+]|x,M\rangle^- = M|x,M\rangle^-. \tag{45}$$

As an example, we try to get the ladder operator formalism of the PACS. The CS expanded in the Fock space is

$$|\psi\rangle_{CS} = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{46}$$

where α is a complex number. Using the coefficients of the CS and Eq.(41), we obtain the ladder operator formalism of the PACS

$$(\hat{N}+1-M)a|\alpha,M\rangle = \alpha(\hat{N}+1)|\alpha,M\rangle. \tag{47}$$

Since the operator $1/(\hat{N}+1)$ is non-zero in the whole Fock space, we get

$$[1-M/(\hat{N}+1)]a|\alpha,M\rangle=\alpha|\alpha,M\rangle. \tag{48}$$

According to the definition of the nonlinear coherent states(NLCSs)[19]

$$f(\hat{N})a|\alpha\rangle_{NLCS}=\alpha|\alpha\rangle_{NLCS,} \tag{49}$$

the PACS is an NLCS as discussed by Sivakumar[20]. Here $f(\hat{N})$ is a nonlinear function of $\hat{N}$.

As another example, we consider a new definition of NBS(NNBS)[21] recently introduce by Barnett,

$$|\eta,M\rangle_{NNBS}=\sum_{n=M}^{\infty}\left[\binom{n}{M}\eta^{M+1}(1-\eta)^{n-M}\right]^{1/2}|n\rangle, \tag{50}$$

It is found that the NNBS and the BS have similar properties if the roles of the creation operator $a^{\dagger}$ and annihilation operator a are interchanged.

Form Eq.(38), the ladder operator formalism of the NNBS is obtained as

$$[\sqrt{\hat{N}+1-M}/(\hat{N}+1)]a|\eta,M\rangle_{NNBS}=\sqrt{1-\eta}|\eta,M\rangle_{NNBS}. \tag{51}$$

As seen from the above equation, the NNBS is an NLCS with the nonlinear function $f(\hat{N})=\sqrt{\hat{N}+1-M}/(\hat{N}+1)$. Eq.(51) has been obtained in our previous paper[22]

C. General quantum states

By the analogous method in obtaining Eq.(37), we obtain the ladder operator formalism of the general state $|\psi\rangle$(Eq.(32)) as

$$[\hat{N}-\frac{C(\hat{N},x)}{C(\hat{N}-1,x)}\sqrt{\hat{N}}a^{\dagger}]|\psi\rangle=0. \tag{52}$$

Multiplying the both sides of the above equation by a from left, we get another form of the ladder operator formalism of the state $|\psi\rangle$,

$$a|\psi\rangle=\frac{C(\hat{N}+1,x)}{C(\hat{N},x)}\sqrt{\hat{N}+1}|\psi\rangle. \tag{53}$$

The algebra involved in the state $|\psi\rangle$ is also GDO algebra with generators

$$\hat{N}, [C(\hat{N},x)/C(\hat{N}-1,x)]\sqrt{\hat{N}}a^\dagger,\ a\sqrt{\hat{N}}C(\hat{N},x)/C(\hat{N}-1,x). \tag{54}$$

The corresponding structure function is $\hat{N}^2 C^2(\hat{N},x)/C^2(\hat{N}-1,x)$.

Using the coefficients of the CS(Eq.(46)), we obtain the well-known ladder operator formalism of the CS from Eq.(53)

$$a|\alpha\rangle_{CS} = \alpha|\alpha\rangle_{CS} \tag{55}$$

The above equation can be naturally obtained by letting $M = 0$ in Eq.(48).

Now we consider the geometric state(GS) which is defined as[23]

$$|\eta\rangle_{GS} = \eta^{1/2}\sum_{n=0}^{\infty}(1-\eta)^{n/2}|n\rangle \tag{56}$$

It is also called Susskind-Glogower phase state, phase eigenstate,and coherent phase state[23] in the literature. Form Eq.(53), we get the ladder operator formalism of the GS

$$a|\eta\rangle_{GS} = \sqrt{1-\eta}\sqrt{N+1}|\eta\rangle_{GS}. \tag{57}$$

Since the operator $\sqrt{N+1}$ is not zero in the whole Fock space, we obtain

$$a/\sqrt{N+1}|\eta\rangle_{GS} = \sqrt{1-\eta}|\eta\rangle_{GS}. \tag{58}$$

This equation shows that the GS is an NLCS with the nonlinear function $1/\sqrt{N+1}$. By setting $M = 0$ in Eq.(51), Eq.(51) reduces to Eq.(58). This fact is easily understood since the NNBS can be viewed as photon-added GS[22].

Further we study another example, the NBS, which is defined as[4]

$$|\eta, M\rangle_{NBS} = \sum_{n=0}^{\infty}(1-\eta)^{M/2}\binom{M+n-1}{n}^{1/2}\eta^{n/2}|n\rangle. \tag{59}$$

From Eq.(53), we get

$$\frac{1}{\sqrt{M+\hat{N}}}a|\eta, M\rangle_{NBS} = \eta^{1/2}|\eta, M\rangle_{NBS}, \tag{60}$$

which shows that the NBS is an NLCS with the nonlinear function $f(\hat{N}) = 1/\sqrt{M+\hat{N}}$. The result is identical to that obtained by us before[24].

It is interesting in investigate the Kerr state(KS) which is defined as[25]

$$|\alpha,\theta\rangle_{KS} = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n \exp[-i\theta n(n-1)]}{\sqrt{n!}}|n\rangle. \tag{61}$$

From Eq.(53), the ladder operator formalism of the KS is obtained as

$$\exp(-2i\hat{N}\theta)a|\alpha,\theta\rangle_{KS} = \alpha|\alpha,\theta\rangle_{KS}. \tag{62}$$

When $\theta = 0$, Eq.(62) naturally reduce to Eq.(55). Eq.(62) show that the KS is an NLCS with the nonlinear function $f(\hat{N}) = \exp(-2i\hat{N}\theta)$. All the states discussed in this section is of one-photon type. We will study two-photon quantum states in next section.

III. TWO-PHOTON QUANTUM STATES

The representative two-photon states are the squeezed vacuum states(SVSs) (defined below) and even/odd coherent states(ECS/OCS)[26]. These states are defined in either even Fock space or odd Fock space.

The squeezed vacuum state is defined as

$$|\xi\rangle_{SVS} = S(\xi)|0\rangle, \tag{63}$$

where $S(\xi) = \exp(\xi K_+ - \xi^* K_-)$ is the squeezing operator and $\xi = r\exp(i\theta)$. Here $K_+ = a^{+2}/2$, $K_- = a^2/2$. The two operators together with the operator $K_0 = \hat{N}/2 + 1/4$ form a su(1,1) Lie algebra.

The representation of su(1,1) on the usual Fock space is completely reducible and decomposes into a direct sum of the even Fock space (S_0) and odd Fock space (S_1)[27],

$$S_j = \text{span}\{||n\rangle_j \equiv |2n+j\rangle|n = 0,1,2,...\},\ j = 0,1. \tag{64}$$

Representations on S_j can be written as

$$
\begin{aligned}
K_+||n\rangle_j &= \sqrt{(n+1)(n+j+1/2)}||n+1\rangle_j, \qquad (65)\\
K_-||n\rangle_j &= \sqrt{(n)(n+j-1/2)}||n-1\rangle_j,\\
K_0||n\rangle_j &= (n+j/2+1/4)||n\rangle_j.
\end{aligned}
$$

The Bargmann index $k = 1/4(3/4)$ for even(odd) Fock space.

It is easily seen that the SQS is define on the even Fock space. For obtaining the expansion of the SQS in the even Fock space, we use the decomposed form of the squeezing operator

$$
S(\xi) = \exp(e^{i\theta}\tanh r\ a^{+2}/2)(\cosh r)^{-(a^+a+1/2)}\exp(-e^{-i\theta}\tanh r\ a^2/2). \qquad (66)
$$

From the above equation and Eq.(63), we obtain the expansion as

$$
|\xi\rangle_{SVS} = (\cosh r)^{-1/2}\sum_{n=0}^{\infty}\sqrt{(2n)!}[e^{i\theta}\tanh r/2]^n/n!||n\rangle_0. \qquad (67)
$$

Now we consider two general states in even/odd Fock space

$$
\begin{aligned}
|x\rangle_e &= \sum_{n=0}^{\infty} C_e(n,x)||n\rangle_0, \qquad (68)\\
|x\rangle_o &= \sum_{n=0}^{\infty} C_o(n,x)||n\rangle_1. \qquad (69)
\end{aligned}
$$

For convenience, we introduce the number operator $\hat{N}_0,\hat{N}_1$ by

$$
\begin{aligned}
\hat{N}_0 &= K_0 - 1/4, \hat{N}_0||n\rangle_0 = n||n\rangle_0, \qquad (70)\\
\hat{N}_1 &= K_0 - 3/4, \hat{N}_1||n\rangle_1 = n||n\rangle_1. \qquad (71)
\end{aligned}
$$

Using the same method in deriving Eq.(37), we obtain the ladder operator formalisms of the states$|x\rangle_{e/o}$,

$$[\hat{N}_0 - \frac{\sqrt{\hat{N}_0} C_e(\hat{N}_0, x)}{C_e(\hat{N}_0 - 1, x)\sqrt{\hat{N}_0 - 1/2}} \frac{a^{+2}}{2}]x\rangle_e \;=\; 0, \tag{72}$$

$$[\hat{N}_1 - \frac{\sqrt{\hat{N}_1} D_o(\hat{N}_1, x)}{D_o(\hat{N}_1 - 1, x)\sqrt{\hat{N}_1 + 1/2}} \frac{a^{+2}}{2}]x\rangle_o \;=\; 0. \tag{73}$$

The above equation can be written in terms of a^2 and N_j,

$$[\frac{C_e(\hat{N}_0 + 1, x)\sqrt{\hat{N}_0 + 1/2}(N_0 + 1)}{C_e(N_0, x)} - \sqrt{\hat{N}_0 + 1}\frac{a^2}{2}]x\rangle_e \;=\; 0, \tag{74}$$

$$[\frac{C_o(\hat{N}_1 + 1, x)\sqrt{N_1 + 3/2}(N_1 + 1)}{C_o(N_1, x)} - \sqrt{\hat{N}_1 + 1}\frac{a^2}{2}]x\rangle_o \;=\; 0. \tag{75}$$

Substituting the coefficients of the SVS into Eq.(72) leads to

$$\frac{1}{\hat{N} + 1} a^2 |\xi\rangle_{SVS} = \exp(i\theta) \tanh r |\xi\rangle_{SVS}. \tag{76}$$

We see that the SVS is the two-photon nonlinear coherent state(TPNLCS) which can be defined as

$$f(\hat{N}) a^2 |\alpha\rangle_{TPNLCS} = \alpha |\alpha\rangle_{TPNLCS}. \tag{77}$$

As another squeezed state, the squeezed first excited state(SFES) is defined as

$$|\xi\rangle_{SFES} = S(\xi)|1\rangle, \tag{78}$$

By using Eq.(66), we get the expansion of the state $|\xi\rangle_{SFES}$

$$|\xi\rangle_{SFES} = (\cosh r)^{-3/2} \sum_{n=0}^{\infty} \sqrt{(2n+1)!}[e^{i\theta} \tanh r/2]^n / n! |n\rangle_0. \tag{79}$$

Similar to the derivation of Eq.(76), we obtain

$$\frac{1}{\hat{N} + 2} a^2 |\xi\rangle_{SVS} = \exp(i\theta) \tanh r |\xi\rangle_{SVS} \tag{80}$$

.

The above equation shows that the SFES is a TPNLCS.

Another type of examples are the even CS(ECS) and odd CS(OCS)[26]

$$
\begin{aligned}
|\alpha\rangle_{ECS} &= 1/\sqrt{\cosh|\alpha|^2}\sum_{n=0}^{\infty}\alpha^{2n}/\sqrt{(2n)!}||n\rangle_{0,}. &(81)\\
|\alpha\rangle_{OCS} &= 1/\sqrt{\sinh|\alpha|^2}\sum_{n=0}^{\infty}\alpha^{2n+1}/\sqrt{(2n+1)!}||n\rangle_{1.} &(82)
\end{aligned}
$$

The ladder operator formalism can be easily obtained as from Eqs(74) and (75)

$$
\begin{aligned}
a^2|\alpha\rangle_{ECS} &= \alpha^2|\alpha\rangle_{ECS}, &(83)\\
a^2|\alpha\rangle_{OCS} &= \alpha^2|\alpha\rangle_{OCS}.
\end{aligned}
$$

This is just what we expected.

As seen from Eqs.(72) and (73), we conclude that the algebra involved in the two general two-photon states are the GDO algebra with generators

$$
\hat{N}_0, \frac{\sqrt{\hat{N}_0}C_e(\hat{N}_0,x)}{C_e(\hat{N}_0-1,x)\sqrt{\hat{N}_0-1/2}}\frac{a^{+2}}{2}, \frac{a^2}{2}\frac{\sqrt{\hat{N}_0}C_e(\hat{N}_0,x)}{C_e(\hat{N}_0-1,x)\sqrt{\hat{N}_0-1/2}}, \tag{84}
$$

and

$$
\hat{N}_1, \frac{\sqrt{\hat{N}_1}D_o(\hat{N}_1,x)}{D_o(\hat{N}_1-1,x)\sqrt{\hat{N}_1+1/2}}\frac{a^{+2}}{2}, \frac{a^2}{2}\frac{\sqrt{\hat{N}_1}D_o(\hat{N}_1,x)}{D_o(\hat{N}_1-1,x)\sqrt{\hat{N}_1+1/2}}. \tag{85}
$$

The corresponding structure functions are

$$
\begin{aligned}
F_e(\hat{N}_0) &= \frac{\hat{N}_0^2 C_e^2(\hat{N}_0,x)}{C_e^2(\hat{N}_0-1,x)},\\
F_o(\hat{N}_1) &= \frac{\hat{N}_1^2 D_o^2(\hat{N}_1,x)}{D_o^2(\hat{N}_1-1,x)}. &(86)
\end{aligned}
$$

IV. CONCLUSIONS

We show that various kinds of states in the field of quantum optics admit ladder operator formalisms and bear GDO algebraic structures. The corresponding structure functions are obtained. As examples we give ladder operator formalisms of the BS, RBS, NBS, NNBS, HGS, PS, RBS, PBPS, GS, GGS and KS. We also consider the two-photon case and get the ladder operator formalisms of two general states defined in the even/odd Fock space . We found that the algebra involved in the two general states are also GDO algebra. The ladder operator formalisms of some special two-photon states, the SVS, SFES, ECS and OCS, are obtained. The SVSs and SFESs are found to be TPNLCSs. In addition, the method proposed here can be applied to the study of ladder operator formalism and algebraic structure of quantum states for su(1,1) and su(2) Lie algebra. The work is in progress.

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